

Time Dilation

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General Time Dilation

The formula for time dilation in Special Relativity is $\Delta t = \gamma \Delta t_0 = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, where v is the velocity of the object relative to the observer's inertial frame, c is the speed of light, Δt is the observed time between the two events, and Δt_0 is the actual (i.e. in the reference frame of the relatively moving object) time between the two events.

1 Time Dilation at the Speed of Light

We see that if we take the limit as v approaches the speed of light from a lesser speed, the observed time approaches infinity:

$$\lim_{v \rightarrow c} \Delta t = \frac{\Delta t_0}{0} \Rightarrow \lim_{v \rightarrow c} \Delta t \rightarrow +\infty$$

2 What Happens When an Object is Going Faster Than c ?

Say we have an object travelling at twice the speed of light:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{4c^2}{c^2}}}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{-3}}$$

Uh-oh! Imaginary numbers! This means that this is impossible in our dimensions of reality.

But what if we ignore that?

$$\Delta t = \frac{\Delta t_0}{\sqrt{3}i}$$

$$\Delta t \approx .577 \Delta t_0 i$$

That means that the time observed between the two events is *less* than the time it took to do them in the moving object's reference frame. Essentially, the event seems to occur *before* it actually happens. To the inertial reference frame (observer), this is time travel.